Python Basics

1. Although most in the industry will be reasonably forgiving when it comes to coding by hand, you should still be comfortable with the syntax of your language of choice.

   (a) Identify and fix the line(s) that have invalid syntax in the following code:

   If you are unsure about a specific line, just write a brief explanation of the error. Assume that incorrect lines do not affect the validity of other lines which may depend on them.

   ```python
   1 include math
   2
   3 int a = 0
   4 b = 0
   5
   6 def function(arg):
   7     return (arg + 2)
   8
   9 c = "hello"
10 sum = b + math.sqrt(c)
11 total = a + (b ** 0.5)
   ```

   (b) Which of the following are keywords or standard functions in Python?

   ```
   import in from class define int str String
   True false where while or not isinstance new
   print range float char bool xor sum
   ```

General Recursion

2. Sally is being plagued by an army of lookalike suiters, each of which presents her with an enticing but unbearable meal upon arrival. The dishes all smell amazing, however, so she can’t help but try each one. A dish will never fail to disappoint her, but fortunately, some of the suiters shared recipes and created identical concoctions. Once Sally tastes a meal once, she can immediately smell out instances of the same dish and send their bearers away.

   (a) Write a Python function that, given a list of “dishes,” returns a list of the rejected dishes in the order they failed to fool her. (e.g. [1, 2, 2, 3, 3, 3] would return [2,3,3] )
(b) If your solution to 2a was iterative, write it recursively; if it was recursive, write it iteratively.

(c) What is the time complexity of your approach? Why?
3. Write a recursive function that reverses a string (e.g. “Racecar” yields “racecaR”).

4. Perform a substitution trace on \texttt{reverse(’Doge’)}.

\textbf{Files/IO}

5. Write a function that takes in a file name, and returns the average size of a word in that file. Assume the files will only have 1 word per line, for example:

\begin{verbatim}
No
soup
for
you!
\end{verbatim}

which has an average length of: 3.25

\textit{Assume a function \texttt{len( str ) which returns the length of a string} is provided.}
Tail Recursion

6. Below is a function that, given a lower bound $i$, upper bound $n$, and function from integers to integers $f$, computes $\sum_{k=i}^{n} f(k)$.

```python
1   def series_sum(i, n, f):
2       if(i == n):
3           return f(n)
4       return f(i) + series_sum(i + 1, n, f)
```

(a) Invoke the function to compute $\sum_{k=1}^{5} k^2$.

(b) Rewrite the `series_sum` function to be tail recursive.

```python
1   def series_sum(i, n, f, s =0):
2       if(i == n):
3           return s + f(n)
4       return series_sum(i + 1, n, f, s + f(i))
```

(c) What is the advantage of the new implementation?

Greedy Algorithms

7. Given that an algorithm is greedy, is it guaranteed to return an optimal solution?
Structures

8. For the sake of this question, you find yourself to be the head programmer under Kim Jong Un’s glorious reign. It also just so happens that a nation-wide track meet is being held today. Thus, the glorious leader has demanded that you write a program to keep track of information relating to all track runners present at the event.

(a) Write a struct called TrackRunner to keep track of each competing runner. You will need to store each runner’s name (a string), age (an int), and fastestTime (an int).

(b) Now write a function to make an individual TrackRunner struct.

(c) The glorious leader has decided that, on this day, no runner named "Joe" may win gold. Given a list of TrackRunner structs, write the function aWinnerIsYou(runners) that returns the runner in the list runners with the fastest time whose name is not “Joe”. Use this function to find the runner’s name, age, and fastest time and print those results.

def aWinnerIsYou(runners):

Stacks and Queues

9. How can you use a stack to check if an input string of exclusively parentheses, brackets, and braces is properly balanced? (e.g. “[]{}” is accepted, but “()” and “[{}“ are not.) Assume that you are provided with a Stack structure that has push(), pop(), and peek() functions.
Searching

10. Given the sorted list [1, 4, 9, 16, 25, 69, 420, 1337], write out the steps that a binary search would take to find the number 69.

Trees

11. (a) If we have a balanced binary search tree containing \( n \) nodes:
   i. What is its height?
   
      ii. How much time would it take to traverse to any of the leaf nodes of this tree?
   
      iii. What’s the worst case search time for an unbalanced search tree?

   (b) How many leaves are in a complete BST containing \( n \) nodes?

   (c) In terms of runtime efficiency, what is the worst possible configuration (e.g. arrangement of nodes and their children) of a binary search tree? Sketch a small example of what such a configuration looks like.

   (d) Based on your answer above, what data structure is such a BST reminiscent of?
12. (a) Write a binary tree program that defines the following:

- a **TreeNode** struct with the given properties:
  - `data` (an object): the data in this node
  - `left`: the **TreeNode** head of the left subtree (or `None`)
  - `right`: the **TreeNode** head of the right subtree (or `None`)
- a **Tree** struct with the given properties:
  - `size` (an int): the size of the binary tree
  - `head` (**TreeNode**): the head node
- a recursive function `is_in_tree(head, element)` that determines whether the binary tree with the given head contains the specified element

(b) Give an example of how you could test your code.

```python
newTree = Tree(3, TreeNode(3, TreeNode(1, None, None), TreeNode(4, None, None)))
print(is_in_tree(newTree.head, 1))  # -> True
print(is_in_tree(newTree.head, 3))  # -> True
print(is_in_tree(newTree.head, 5))  # -> False
```
Heaps

13. For a binary heap containing \( n \) elements, what is the maximum number of swaps occurring after an insert operation?

14. Given a node in an array-based binary heap at index \( i \), where are the indices of both its children? What is the index of its parent?

15. Run a heap sort in reverse order on the following list: \([3,5,1,3,2,7,9]\), showing the heap at each stage. Be sure to heapify the list first.
16. Fill in the table for the asymptotic running time of each sorting algorithm.

<table>
<thead>
<tr>
<th></th>
<th>Best</th>
<th>Worst</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merge sort</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
</tr>
<tr>
<td>Quicksort</td>
<td>$O(n \log(n))$</td>
<td>$O(n^2)$</td>
<td>$O(n \log(n))$</td>
</tr>
<tr>
<td>Heap sort</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
<td>$O(n \log(n))$</td>
</tr>
</tbody>
</table>

17. Below is Python code for a function that performs an insertion sort and prints `data` after each iteration of the `for` loop.

```python
1 def insertion_sort( data ):
2     for marker in range( 1, len( data ) ):
3         val = data[marker]
4         i = marker
5         while i > 0 and data[i-1] > val:
6             data[i] = data[i-1]
7             i -= 1
8         data[i] = val
9         print( data )
```

(a) Write out what the function will print for the input list: `[3,2,7,1]`.

(b) What is the sort algorithm’s time complexity?

18. In what scenario does Quicksort experience its worst-case time complexity? You may assume that we always pick the first element as the pivot.

19. What causes Quicksort to run so slowly on the input you describe in the last question?

20. In Quicksort, why should we select a random pivot value, rather than always pivoting on, for example, the first or last element?
21. Show the stages of a merge sort and a quicksort on the following list: [3,5,1,3,2,7,9]. Be sure to identify your pivot.